UNIFICATION OF SPINS AND CHARGES IN GRASSMANN SPACE?

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ABSTRACT

In a space of d ($d > 5$) ordinary and d Grassmann coordinates, fields manifest in an ordinary four-dimensional subspace as spinor $(1/2, 3/2)$, scalar, vector or tensor fields with the corresponding charges , according to two kinds of generators of the Lorentz transformations in the Grassmann space. Vielbeins and spin connections define gauge fields-gravitational and Yang-Mills.

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1. Introduction. In supersymmetric theories [1] fermions enter into theories through spinor charges, while electromagnetic, weak and colour charges are defined by generators of internal (additional) groups. Vierbeins and spin connections, which define gravitational fields, transform vectors from freely falling coordinate systems to external coordinate systems. Modern Kaluza-Klein theories with fermions try to define all gauge fields with vielbeins and spin connections [2], while they connect charges with Killing vectors which determine symmetry groups and cause a compactification of extra dimensions [2] .

In this letter we present the theory in which *all internal degrees of freedom of particles* and *fields -spins* and *charges* -are defined by *two kinds* of *generators* of the *Lorentz transformations* in the *Grassmann* d-dimensional *space* (forming the algebra of the Lorentz group $SO(1, d-1)$). The generators of a spinorial character define spinors: generators with indices of the four dimensional subspace define spins of spinors, while the generators with indices of higher dimensions define charges of these spinors. All gauge fields - gravitational as well as Yang-Mills including electrodynamics - are defined by (super)vielbeins, whose spins and charges are determined by the generators of the Lorentz group of a vectorial charater.

In our theory the space has d ordinary $(\{x^a\}, commuting)$ and d Grassmann $(\{\theta^a\}, anticommuting, \theta^a\theta^b + \theta^b\theta^a = 0)$ coordinates.

Both kinds of generators of translations in the Grassmann part of the space have an odd Grassmann character. While those of a vectorial character form the Grassmann odd Heisenberg algebra, form those of a spinorial character the Clifford

algebra. Generators of translations and the Lorentz transformations of both kinds are after the canonical quantization of coordinates the differential operators in the Grassmann space, and the Grassmann space (and accordingly the ordinary space) has to have at least 5 dimensions in order that the Dirac γ^a operators are the Grassmann even differential operators .

Since the Lorentz algebra $SO(2n+1)$ has a regular maximal subalgebra $SO(2m)\times SO(2n-2m+1)$, defines the choice $n=7$ and $m=5$, for example, the Lorentz $SO(1,4)$ subalgebra, generators of which determine scalars, spinors, vectors and tensors in the four dimensional part of the space, while generators of $SO(10)$ subalgebra define the electromagnetic, weak and colour charges (in the corresponding fundamental and adjoint representations). It is the dependence of fields on Grassmann coordinates, which determines spins and charges of all fields.

Vector space spanned over a d dimensional Grassmann space has the dimension 2^d . Half of vectors have an even, half of vectors have an odd Grassmann character, demonstrating the supersymmetry of the theory . The canonical quantization of fields quantizes the former to bosons, the later to fermions[3].

Generators of translations and the Lorentz transformations in the ordinary and the Grassmann space form the super-Poincaré algebra[3]. The super-Pauli-Ljubanski vector can be defined as a generalization of the Pauli-Ljubanski vector with an odd Grassmann character[3]. It defines spinor charges.

(Super)vielbeins, depending on the ordinary and the Grassmann coordinates (and connecting (super)vectors of a freely falling coordinate system to (super)vectors of an external coordinate system[3]), define all gauge fields. A spin connection appearing as a (super)partner of a vielbein has an odd Grassmann character and describes a fermionic part of a gravitational field.

2. A particle in a freely falling coordinate system. Since the space has two kinds of coordinates, commuting ones $\{x^a\}$ and anticommuting ones $\{\theta^a\}$, the geodesics is determined with both kinds of coordinates and with two parameters: one of a Grassmann even (τ) another of a Grassmann odd (ξ) character : $X^a = X^a(x^a,\theta^a,\tau,\xi)$. They are called supercoordinates[3,4]. We define the dynamics of a particle by choosing the (simplest) action

$$I=\frac{1}{2}\int d\tau d\xi E E^i_A \partial_i X^a E^j_B \partial_j X^b \eta_{ab}\eta^{AB}, \tag{2.1}$$

where $\partial_i := (\partial_\tau, \overrightarrow{\partial}_\xi), \tau^i=(\tau,\xi)$, while E^i_A determines a metric on a two dimensional superspace τ^i , $E=det(E^i_A)$. We choose $\eta_{AA}=0, \eta_{12}=1=\eta_{21}$, while η_{ab} is the Minkowski metric with the diagonal elements $(1,-1,-1,-1,...,-1)$. The action is invariant under the Lorentz transformations in the d (ordinary and Grassmann) space and under general coordinate transformations in a two dimansional space τ^i.

Taking into account that either x^a or θ^a depend on an ordinary time parameter τ and that $\xi^2 = 0$, geodesics can be described (a special choice) as a polynomial of ξ as follows: $X^a = x^a + \varepsilon\xi\theta^a$. We choose ε^2 to be equal either to $+i$ or to $-i$ so that it defines two possible combinations for supercoordinates. Accordingly we choose also the metric ${E^i}_A$[3,5]: ${E^1}_1 = 1, {E^1}_2 = -\varepsilon M, {E^2}_1 = \xi, {E^2}_2 = N - \varepsilon\xi M$, with N and M a Grassmann even and odd parameter, respectivelly. We write $\dot{A} = \frac{d}{d\tau}A$, for any A.

In the Grassmann space the left derivatives have to be distinguished from the right derivatives, due to the anticommuting nature of coordinates[5]. We shall make use of left derivatives only, defined as follows: $\frac{\overrightarrow{\partial}}{\partial\theta^a}\theta^b f = \delta^b_a f - \theta^b \frac{\overrightarrow{\partial}}{\partial\theta^a} f$.

It turns out[3] that for a particle whose geodesics is defined in the space of ordinary and Grassmann coordinates, the Grassmann coordinates are proportional to their conjugate momenta $p^\theta_a := \frac{\overrightarrow{\partial} L}{\partial\dot{\theta}^a} = \epsilon^2\theta^a$. Here L is the Lagrange function which follows from the action(2.1) if the integral over $d\xi$ is performed[3]. It is appropriatelly to define generalized coordinates[3]

$$\tilde{a}^a := i(p^{\theta a} - i\theta^a), \;\; \tilde{\tilde{a}}^a := -(p^{\theta a} + i\theta^a). \tag{2.2}$$

The generators of the Lorentz transformations for the action(2.1), which are

$$M^{ab} = L^{ab} + S^{ab} \; , \; L^{ab} = x^a p^b - x^b p^a \; , \; S^{ab} = \theta^a p^{\theta b} - \theta^b p^{\theta a}, \tag{2.3}$$

show that parameters of the Lorentz transformations are the same in both spaces. The generators may be written with respect to operators $\tilde{a}^a$ and $\tilde{\tilde{a}}^a$

$$S^{ab} = \tilde{S}^{ab} + \tilde{\tilde{S}}^{ab} \; , \; \tilde{S}^{ab} = -\frac{i}{4}(\tilde{a}^a\tilde{a}^b - \tilde{a}^b\tilde{a}^a) \; , \; \tilde{\tilde{S}}^{ab} = -\frac{i}{4}(\tilde{\tilde{a}}^a\tilde{\tilde{a}}^b - \tilde{\tilde{a}}^b\tilde{\tilde{a}}^a). \tag{2.3a}$$

The choice of ε makes either $\tilde{\tilde{a}}^a(\varepsilon^2 = -i)$ or $\tilde{a}^a(\varepsilon^2 = i)$ equal to zero and accordingly either $\tilde{\tilde{S}}^{ab} = 0$ or $\tilde{S}^{ab} = 0$.

In the canonical procedure the Poisson brackets follow, treating equivalently the ordinary and the Grassmann space [3]:

$$\{B, A\}_p = -\frac{\partial A}{\partial x^a}\frac{\partial B}{\partial p_a} + \frac{\partial A}{\partial p_a}\frac{\partial B}{\partial x^a} - (\frac{\overrightarrow{\partial A}}{\partial\theta^a}\frac{\overrightarrow{\partial B}}{\partial p^\theta_a} + \frac{\overrightarrow{\partial A}}{\partial p^\theta_a}\frac{\overrightarrow{\partial B}}{\partial\theta^a})(-1)^{n_A}, \tag{2.4}$$

where n_A is either one or two depending on whether A has on odd or an even Grassmann character, respectively.

In the quantization procedure[3] $-i\{A, B\}_p$ goes to either a commutator or to an anticommutator, according to the Poisson brackets (2.4). The operators $\theta^a, p^{\theta a}$

(in the coordinate representation they become $\theta^a \longrightarrow \theta^a,\ p^\theta_a \longrightarrow -i\frac{\overrightarrow{\partial}}{\partial\theta^a}$) fulfil the Grassmann odd Heisenberg algebra, while the operators
$\tilde{a}^a$and $\tilde{\tilde{a}}^a$ fulfil the Clifford algebra:

$$\{\theta^a, p^{\theta b}\} = -i\eta^{ab},\ \{\tilde{a}^a, \tilde{a}^b\} = 2\eta^{ab} = \{\tilde{\tilde{a}}^a, \tilde{\tilde{a}}^b\}, \tag{2.5}$$

with $\{\tilde{a}^a, \tilde{\tilde{a}}^b\} = 0 = \{\tilde{S}^{ab}, \tilde{\tilde{S}}^{cd}\}$ and $\tilde{S}^{ab} = -\frac{i}{4}[\tilde{a}^a, \tilde{a}^b]_-, \tilde{\tilde{S}}^{ab} = -\frac{i}{4}[\tilde{\tilde{a}}^a, \tilde{\tilde{a}}^b]_-$.

Either L^{ab} or S^{ab} or $\tilde{S}^{ab}$ or $\tilde{\tilde{S}}^{ab}$ form the Lie algebra of the Lorentz group.

It appears[3] that S^{ab} define the adjoint representations while $\tilde{S}^{ab}$ and $\tilde{\tilde{S}}^{ab}$ define the fundamental representations of the Lorentz group.

The constraints which follow from the action(2.1) lead to the Dirac and to the Klein-Gordon equation

$$p^a\tilde{a}_a|\tilde{\Psi}>= 0\ ,\ p^a p_a|\tilde{\Psi}>= 0,\ with\ p^a\tilde{a}_a p^b\tilde{a}_b = p^a p_a. \tag{2.6}$$

Since the operator $\tilde{a}^a$ (which is a differential operator in the Grassmann space) has an odd Grassmann character, it can not be recognized as the Dirac $\tilde{\gamma}^a$ operator. The dimension of the space d has therefore to be at least five ($d \geq 5$) (which means at least 5 odinary and 5 Grassmann coordinates) in order that the generators of the Lorentz transformations $\tilde{S}^{5m}, m = 0, 1, 2, 3$ can be recognized as the Dirac γ^m operators of an even Grassmann character

$$\tilde{\gamma}^m = -\tilde{a}^5\tilde{a}^m = -2i\tilde{S}^{5m}\ ,\ m = 0, 1, 2, 3. \tag{2.7}$$

It can be checked that in the four-dimensional subspace $\tilde{\gamma}^m$ fulfil the Clifford algebra $\{\tilde{\gamma}^m, \tilde{\gamma}^n\} = \eta^{mn}$, while $\tilde{S}^{mn} = -\frac{i}{4}[\tilde{\gamma}^m, \tilde{\gamma}^n]_-$. The operator $\tilde{\Gamma} = i\tilde{a}^0\tilde{a}^1\tilde{a}^2\tilde{a}^3 = i\tilde{\gamma}^0\tilde{\gamma}^1\tilde{\gamma}^2\tilde{\gamma}^3$ is one of the two Casimir operators of the Lorentz group $SO(1, 3)$. We have seen, however, that not $SO(1, 3)$ but (at least) $SO(1, 4)$ is needed to properly define the Dirac algebra.

In the case that $d = 5$ and $< \tilde{\psi}|p^5|\tilde{\psi} >= m$ it follows

$$(\tilde{a}^m p_m - \tilde{a}^5 p^5)|\tilde{\psi} >= 0 = (\tilde{\gamma}^m p_m - m)|\tilde{\psi} >\ ,\ m = 0, 1, 2, 3. \tag{2.8}$$

We show in the references[3] four Dirac four-spinors (the polynoms of θ^a) which fulfil the eqs.(2.8) if $m \neq 0$ and four Weyl four-spinors which fulfil the eqs.(2.8) if $m = 0$. We define there the super-Pauli-Ljubanski vector which generates spinor charges. We show also two scalars, two-three vectors and two four-vectors of an even Grassmann character, eigenvectors of S^{ab} (of a vectorial character).

For large enough d not only the generators of the Lorentz transformations (of a spinorial character) in the Grassmann space define (after the canonical quantization of coordinates) spinorial degrees of freedom of a particle field in the four dimensional subspace, they define also quantum numbers of these fields which may

be recognized as electromagnetic, weak and colour charges. For $d = 15$, for example, we can express the generators of the groups $U(1), SU(2)$ and $SU(3)$ in terms of the generators of the Lorentz group $\tilde{S}^{ab}$ with $a, b = 6, 7, ..., 15$ while $SO(5)$ remains to define the spinorial degrees of freedom in the four dimensional subspace. We find the fundamental representations of the corresponding Casimir operators as functions of θ^a determining isospin doublets, colour triplets and electromagnetic singlets. Due to the limited size of this letter we shall present this results elsewhere.

3. A particle in (Kaluza-Klein) gauge fields. We find the dynamics of a point particle in a gravitational field by transforming vectors from a freely falling to an external coordinate system. To do this, vielbeins have to be introduced[1,3]. In our case vielbeins $\mathbf{e}^{ia}{}_{\mu}$, depend on ordinary and on Grassmann coordinates, as well as on two types of parameters $\tau^i = (\tau, \xi)$. Due to two kinds of derivatives ∂_i there are two kinds of vielbeins. The index a refers to a freely falling coordinate system (a Lorentz index), the index μ refers to an external coordinate system (an Einstein index). Vielbeins with the Lorentz index smaller than five will determine ordinary gravitational fields, those with the Lorentz index higher than four will define Yang-Mills fields. Spin connections appear in our theory as (a part of) Grassmann odd fields.

We write the transformation of vectors as follows

$$\partial_i X^a = \mathbf{e}^{ia}{}_{\mu} \partial_i X^{\mu} \ , \ \partial_i X^{\mu} = \mathbf{f}^{i\mu}{}_{a} \partial_i X^a \ , \ \partial_i = (\partial_\tau, \partial_\xi). \tag{3.1}$$

From eq.(3.1) it follows that

$$\mathbf{e}^{ia}{}_{\mu} \mathbf{f}^{i\mu}{}_{b} = \delta^a{}_b \ , \ \mathbf{f}^{i\mu}{}_{a} \mathbf{e}^{ia}{}_{\nu} = \delta^{\mu}{}_{\nu}. \tag{3.2}$$

Again we make a Taylor expansion of vielbeins with respect to ξ

$$\mathbf{e}^{ia}{}_{\mu} = e^{ia}{}_{\mu} + \varepsilon \xi \theta^b e^{ia}{}_{\mu b} \ , \ \mathbf{f}^{i\mu}{}_{a} = f^{i\mu}{}_{a} - \varepsilon \xi \theta^b f^{i\mu}{}_{ab} \ , \ i = 1, 2. \tag{3.3}$$

Both expansion coefficients depend again on ordinary and on Grassmann coordinates. Since $e^{ia}{}_{\mu}$ have an even Grassmann character it will describe the spin 2 part of a gravitational field. The coefficients $\varepsilon \theta^b e^{ia}{}_{\mu b}$ have an odd Grassmann character (ε is again the complex constant) and are therefore candidates for spinorial part of a gravitational field. We shall see that they define spin connections[1,3].

From eqs(3.2) and (3.3) it follows that

$$e^{ia}{}_{\mu} f^{i\mu}{}_{b} = \delta^a{}_b \ , \ f^{i\mu}{}_{a} e^{ia}{}_{\nu} = \delta^{\mu}{}_{\nu} \ , \ e^{ia}{}_{\mu b} f^{i\mu}{}_{c} = e^{ia}{}_{\mu} f^{i\mu}{}_{cb} \ , \ i = 1, 2. \tag{3.2a}$$

We find metric tensor $\mathbf{g}^i_{\mu\nu} = \mathbf{e}^{ia}{}_{\mu} \mathbf{e}^i_{a\nu}$, $\mathbf{g}^{i\mu\nu} = \mathbf{f}^{i\mu}{}_{a} \mathbf{f}^{i\nu a}, i = 1, 2$, with an even Grassmann character and the properties $\mathbf{g}^{i\mu\sigma} \mathbf{g}^i_{\sigma\nu} = \delta^{\mu}{}_{\nu} = g^{i\mu\sigma} g^i_{\sigma\nu}$, with $g^i_{\mu\sigma} = e^{ia}{}_{\mu} e^i{}_{a\sigma}$.

We find from eq.(3.1) that vectors in a freely falling and in an external coordinate system are connected as follows: $\dot{x}^a = e^{1a}{}_\mu \dot{x}^\mu$, $\dot{x}^\mu = f^{1\mu}{}_a \dot{x}^a$, $\theta^a = e^{2a}{}_\mu \theta^\mu$, $\theta_\mu = f^{2\mu}{}_a \theta^a$, and $\dot{\theta}^a = e^{1a}{}_\mu \dot{\theta}^\mu + \theta^b e^{1a}{}_{\mu b} \dot{x}^\mu = (e^{2a}{}_\mu \theta^\mu)^{\cdot} = e^{2a}{}_{\nu,\mu_x} \dot{x}^\mu \theta^\nu + e^{2a}{}_\mu \dot{\theta}^\mu + \dot{\theta}^\mu \overrightarrow{e^{2a}}{}_{\nu,\mu_\theta} \theta^\nu$.

We use the notation $e^{2a}{}_{\nu,\mu_x} = \frac{\partial}{\partial x^\mu} e^{2a}{}_\nu$, $\overrightarrow{e^{2a}}{}_{\nu,\mu^\theta} = \frac{\overrightarrow{\partial}}{\partial \theta^\mu} e^{2a}{}_\nu$.

The above equations define the following relations among fields

$$e^{2a}{}_{\mu b} = 0 \ , \ \overrightarrow{e^{2a}}{}_{\nu,\mu_\theta} \theta^\nu = e^{1a}{}_\mu - e^{2a}{}_\mu \ , \ e^{1a}{}_{\mu b} = e^{2a}{}_{\nu,\mu_x} f^{2\nu}{}_b, \tag{3.4}$$

which means that a point particle with a spin sees a spin connection $\theta^b e^{ia}{}_{\mu b}$ related to a vielbein $e^{2a}{}_\nu$.

Rewritting the action(2.1) in terms of an external coordinate system according to eqs.(3.1), using the Taylor expansion of supercoordinates X^μ and superfields $\mathbf{e}^{ia}{}_\mu$ and integrating the action over the Grassmann odd parameter ξ the action

$$I = \int d\tau \{ \frac{1}{N} g^1_{\mu\nu} \dot{x}^\mu \dot{x}^\nu - \epsilon^2 \frac{2M}{N} \theta_a e^{1a}{}_\mu \dot{x}^\mu + \varepsilon^2 \frac{1}{2} (\dot{\theta}^\mu \theta_a - \theta_a \dot{\theta}^\mu) e^{1a}{}_\mu + \varepsilon^2 \frac{1}{2} (\theta^b \theta_a - \theta_a \theta^b) e^{1a}{}_{\mu b} \dot{x}^\mu \}, \tag{3.5}$$

defines the two momenta of the system

$$p_\mu = \frac{\partial L}{\partial \dot{x}^\mu} = p_{0\mu} + \varepsilon^2 \theta^a \theta^b e^1_{a\mu b}, \ \ p^\theta_\mu = \varepsilon^2 \theta_a e^{1a}{}_\mu = \varepsilon^2 (\theta_\mu + \overrightarrow{e}^{2a}{}_{\nu,\mu_\theta} e^2{}_{a\alpha} \theta^\nu \theta^\alpha). \tag{3.6}$$

Here $p_{0\mu}$ are the covariant (canonical) momenta of a particle. For $p^\theta_a = p^\theta_\mu f^{1\mu}{}_a$ it follows that p^θ_a is proportional to θ_a. For a choice $\varepsilon^2 = -i$, $\tilde{a}_a = i(p^\theta_a - i\theta_a)$, while $\tilde{\tilde{a}}_a = 0$. In this case we may write

$$p_{0\mu} = p_\mu - \frac{1}{2} \tilde{S}^{ab} e^1_{a\mu b} = p_\mu - \frac{1}{2} \tilde{S}^{ab} \omega_{ab\mu} \ , \ \omega_{ab\mu} = \frac{1}{2} (e^1_{a\mu b} - e^1_{b\mu a}), \tag{3.6a}$$

which is the ussual expression for the covariant momenta in gauge gravitational fields[1]. One can find the two constraints

$$p^\mu_0 p_{0\mu} = 0 = p_{0\mu} f^{1\mu}{}_a \tilde{a}^a. \tag{3.7}$$

In the quantization procedure the two constraints of eqs.(3.7) $p_{0\mu} f^{1\mu}{}_a \tilde{a}^a p_{0\nu} f^{1\nu}{}_b \tilde{a}^b = 0 = p_{0\mu} f^{1\mu}{}_a \tilde{a}^a$ have to be symmetrized properly, due to the fact that fields depend on ordinary and Grassmann coordinates, in order that the Klein-Gordon and the Dirac equations in the presence of gravitational fields follow correspondingly.

To see how Yang-Mills fields enter into the theory, the Dirac equation (eq.(3.7), after quantizing it) has to be rewritten in terms of fields which determine the gravitation in the four dimensional subspace and of those fields which determine the

gravitation in higher dimensions. This should be done by taking into account the compacification of the space and looking for states of a particle field and gravitational fields. This is an ambitious project. In this letter, we shall limit ourselves on the supposition that a system manifests itself in the way that only some components of fields are different from zero and that they depend on Grassmann coordinates (which determine spins and charges of fields) and on $x^\alpha,\ \alpha = 0, 1, 2, 3$ (this should follow from the periodic boundary conditions in a properly compactified ordinary space, if expectation values of $p^a = 0$, for $a \geq 6$) while $m = -p_5 (e^{15}{}_5)^{-1}$. We find

$$\tilde{\gamma}^a f^{1\mu}{}_a p_{0\mu} = \tilde{\gamma}^m f^{1\alpha}{}_m (p_\alpha - \frac{1}{2}\tilde{S}^{mn}\omega_{mn\alpha} + A_\alpha) + m, \tag{3.8}$$

where $A_\alpha = \tilde{\tau}^i A^i_\alpha$, with $\tilde{\tau}^i A^i_\alpha = \frac{1}{2}\alpha^{ihk}\tilde{S}^{hk}\beta^{ilm}\omega_{lm\alpha}, h, k, l, m = 6, 7, 8, ..d$, where α^{ihk} and β^{ihk} are two matrices. According to the subalgebra of the Lie algebra of the Lorentz group $SO(d-5)$, $\tilde{\tau}^i$ may form the appropriate algebra for the desired charges. To obtain eq.(3.8) we require that $e^5{}_\mu = 0 = e^m{}_\sigma$, with $m = 0, 1, 2, 3, \sigma = 6, 7, 8, ..d$.

In eq.(3.8) the fields $\omega_{hk\alpha}$ determine all Yang-Mills fields, including electromagnetic ones. In the case that $e^5{}_\alpha$ is not equal to zero, an additional term occurs: $\tilde{\gamma}^m f^\alpha{}_m A_\alpha$, with $p_5 f^5{}_m = f^\alpha{}_m A_\alpha$, with the properties of an electromagnetic field[3]. It brings, however, wrong magnetic moments of charged particles into the theory, unless all charged particles are made out of very heavy constituents, since the mass and the electromagnetic charge of a particle are related in this case. (This is the known unsolved problem of the Kaluza-Klein theory.)

A torsion and a curvature follow from the Poisson brackets $\{p_{0a}, p_{0b}\}_p$, with $p_{0a} = f^{1\mu}{}_a(p_\mu - \frac{1}{2}\tilde{S}^{cd}\omega_{cd\mu})$.

We find

$$\{p_{0a}, p_{0b}\}_p = -\frac{1}{2}S^{cd}R_{cdab} + p_{0c}T^c{}_{ab}, \tag{3.10}$$

$$R_{cdab} = f^{1\mu}{}_{[a} f^{1\nu}{}_{b]} (\omega_{cd\nu,\mu^x} + \omega_c{}^e{}_\mu \omega_{ed\nu} + \overrightarrow{\omega}_{cd\mu, f^\theta}\theta^e \omega_e{}^f{}_\nu),$$

$$T^c{}_{ab} = e^{1c}{}_\mu (f^{1\nu}{}_{[b} f^{1\mu}{}_{a],\nu} + \omega_{e\nu}{}^d \theta^e f^{1\nu}{}_{[b} \overrightarrow{f^{1\mu}}{}_{a],d^\theta}),$$

with $A_{[a}B_{b]} = A_a B_b - A_b B_a$. It has to be pointed out that the Poisson brackets $\{p_{0\mu}, p_{0\nu}\}_p$ can be written in terms of the odd Grassmann fields $\Psi^a{}_\mu = \theta^b \omega_{ab\mu}$ as well

$$\{p_{0\mu}, p_{0\nu}\}_p = \frac{i}{2}\tilde{a}^a \mathcal{D}_{[\mu}\Psi_{a\nu]},\ \ \mathcal{D}_\mu \Psi_{a\nu} = \Psi_{a\nu,\mu} + \frac{i}{2}\omega_{cd\mu}\tilde{S}^{cd}\Psi_{a\nu}, \tag{3.10a}$$

where $\mathcal{D}_\mu$ appears as a *covariant derivative of a spinor- vector field* $\Psi^a{}_\mu$.

If the action for a free gravitational field is $I = \int d^d x d^d\theta \omega \mathcal{L}$, where ω is a scalar density in the Grassmann space[3], the Lagrange density $\mathcal{L}$ includes $det(e^{1a}{}_\mu)R$,

$R = R^{ab}{}_{ab}$, or (and) $det(e^{1a}{}_{\mu})$ $T^a{}_{cd}T^{cd}{}_a$ as well as terms with $\mathcal{D}_{[\mu}\Psi_{a\nu]}$ combined with $\tilde{\gamma}^{\rho}$ into Rarita- Schwinger like terms[6].

4.Conclusion. The theory in which the space has d ordinary and d Grassmann coordinates possesses supersymmetry and enables the canonical quantization of coordinates and fields. In this theory, spins and charges of spinor fields and gauge fields (gravitational, Yang-Mills and electromagnetic) are defined by two kinds of generators of the Lorentz transformations in the Grassmann space: those of the spinorial character define properties of spinorial fields, while those of the vectorial character define properties of gauge fields. The generators with indices higher than five define charges of particles and fields, those with indices smaller or equal to five define spins of particles and fields. Spin connections have properties of the Rarita-Schwinger field. All gauge fields -gravitational and Yang-Mills - appear through vielbeins and spin connections demonstrating their unification.

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1 J. Wess and J. Bagger,*Supersymmetry and Supergravity*, Princeton Series in Physics (Princeton University Press, Princeton, New Jersey,1983)

2 M.J. Duff, Nucl. Phys. **B 219**, 389 (1983) , M.J. Duff, B.E.W. Nilsson and C.N. Pope, Phys. Lett. **B 139**, 154 (1984)

3 N. Mankoč -Borštnik, Phys.Lett. **B 292**, 25 (1992), Il nuovo Cimento **A 105**, 1461 (1992) , Journ. of Math. Phys. **34**, 8 (1993), Int. Journal of Modern Phys. **A 9**, 1731 (1994), IJS.TP.93/15 (submitted to Phys. Letters), To appear in Proceedings of the Minsk conference Quantum Systems, New Trends and Methods, Minsk, May,1994

4 H. Ikemori, Phys.Lett. **B 199**, 239 (1987)

5 F.A. Berezin and M.S. Marinov, *The Methods of Second Quantization*, Pure and Applied Physics (Accademic Press, New York, 1966)

6 D. Lurié, *Particles and Fields*, Interscience Publishers (John Wiley and Sons, New York 1968)